MATHEMATICS SOLUTION
(CBCGS SEM – 4 DEC 2019)
BRANCH – EXTC ENGINEERING

1 a) If A = $\begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$, then find the eigen value of $6A^{-1} + A^3 + 2I$ **(5)**

Ans :- $$\begin{vmatrix} 2-\lambda & 4 \\ 0 & 3-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(3-\lambda) - 0 = 0$$

$$6 - 2\lambda - 3\lambda + \lambda^2 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda = 2, 3$$

Eigen Value for $6A^{-1} + A^3 + 2I$

$$= 6\left[\frac{1}{\lambda}\right] + \lambda^3 + 2$$

For $\lambda = 2$ For $\lambda = 3$

$$= 6\left[\frac{1}{2}\right] + 2^3 + 2 \qquad = 6\left[\frac{1}{3}\right] + 3^3 + 2$$

$$= 13 \qquad = 31$$

$\therefore$ Eigen values of $6A^{-1} + A^3 + 2I$ are 13 and 31.

1 b) Determine whether the given vectors u = (-4, 6, -10, 1) &v = (2, 1, -2, 9) are orthogonal with repeat to the euclidian inner product. **(5)**

Ans : $u = (-4, 6, 10, 1)$

$v = (2, 1, -2, 9)$

$u.v = (-4 \times 2) + (6 \times 1) + (-10 \times -2) + (1 \times 9)$

$= -8 + 6 + 20 + 9$

$= 27$

For orthogonal $u.v = 0$

$\therefore$ u & v are not orthogonal.

1 c) The probability mass function of a random variable X is zero except at the points X = 0, 1, 2 . At these points it has the values $P\ (0)\ =\ 3c^3,\ \ P\ (1) = 4c - 10c^2,\ P(2) = 5c - 1.$

(i) **Determine C,**

(ii) **Find$P(X < 1),\ P(1 < X \leq 2),\ P(0 < X \leq 2)$.** **(5)**

Ans :- Since $\sum p_i = 1,\quad we\ have,\quad P(0) + P(1) + P(2) = 1$

$$\therefore\ 3c^3 - 10c^2 + 4c + 5c - 1 = 1 \qquad \therefore\ 3c^3 - 10c^2 + 9c - 2 = 0$$

$$(3c - 1)(c - 2)(c - 1) \qquad \therefore\ c = 1/3$$

(The other values are not admissible. Why ?)

$\therefore$ The probability distribution ix

X	0	1	2
P (X = x)	1/9	2/9	2/3

$\therefore\ P(X < 1) = P(X = 0) = \frac{1}{9}\ ;\quad P(1 < X \leq 2) = P(X = 2) = \frac{2}{3};$

$P(0 < X \leq 2) = P(X = 1) + P(X = 2) = \frac{2}{9} + \frac{2}{3} = \frac{8}{9}.$

1 d) Evaluate $\int_c \frac{z+6}{z^2-4}dz$ where C is the circle

(i) $|z| = 1.$

(ii) $|z - 2| = 1$ **(5)**

Ans :- Now $z^2 - 4 = 0\quad gives\ (z + 2)(z - 2) = 0\qquad \therefore z = -2, 2.$

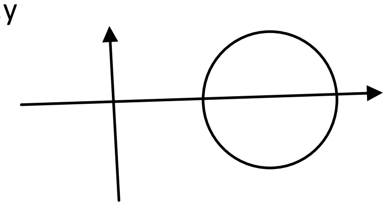
(i) The circle $|z| = 0$ has centre at the origin and radius 1. The points $z = -2, 2$ Lie outside the circle. Hence, f (z) = $\frac{z+6}{z^2-4}$is analytic inside C.

$\therefore$ By Cauchy's integral formula

$$\int_c f(z)dz = 0 \qquad \therefore \int_c \frac{z\ +\ 6}{z^2 - 4}\ dz = 0$$

(ii) The circle $|z-2|=1$ has centre at (2, 0) and radius 1. The point (2, 0) lies inside C and the point (-2, 0) lies outside C. Hence, we write.y

$$\int_c \frac{z+6}{(z-2)(z+2)}\, dz = \int_c \frac{(z+6)/(z+2)}{z-2}\, dz$$ -1 0 (2, 0) x



Now, $f(z) = \frac{z+6}{z+2}$ is analytic in and on C and the point $z = 2$ lies inside C.

$\therefore$ By Cauchy's integral formula

$$\int_c \frac{z+6}{z^2-4}\, dz = \int_c \frac{(z+6)/(z+2)}{z-2}\, dz = 2\pi\, i\, f(z_0)$$

Where, $f(z) = \frac{z+6}{z+2}$ and $z_0 = 2$

Now, $f(z_0) = \frac{2+6}{2+4} = \frac{8}{4} = 2$

$$\therefore \int_c \frac{z+6}{z^2-4}\, dz = 2\pi\, i\,(2) = 4\pi\, i$$

2 a) Using Rayeeigh – Rite method, solve the bound any value problem using a 2 degree polynomial as initial condition. **(6)**

$$I = \int_0^1 2xy + y^2 - (y^1)^2\, dx \quad 0 \le x \le 1$$
$$y(0) = y(1) = 0$$

Ans : **Step I** :$F = 2xy + y^2 - (y^1)^2$

Step II : Assume trail Solution as

$$\bar{y}(x) = c_0 + c_1 x + c_2 x^2$$

Subs $x = 0$

$\bar{y}(0) = c_0 + 0 + 0$ subs initial cond. here to find unknowns

$0 = c_0$

Subs x = 1

$$\bar{y}(1) = c_0 + c_1 + c_2$$
$$\downarrow\downarrow$$
$$0 = 0 + c_1 + c_2$$
$$c_1 + c_2 = 0$$
$$c_2 = -c_1$$

$$\therefore \ \bar{y}(x) = c_1 x - c_1 x^2 = c_1 x(1-x)$$
$$\bar{y}'(x) = c_1 - 2c_1 x \quad = c_1(1-2x)$$

Step III: $I = \int_0^1 2xy + y^2 - (y^1)^2 \, dx$

$$= \int_0^1 2x\left(c_1 x(1-x)\right) + \left(c_1 x(1-x)\right)^2 - \left(c_1(1-2x)\right)^2$$
$$= c_1 \int_0^1 2(x^2 - x^3) + c_1(x^2 - 2x^3 + x^4 - (1 - 4x + 4x^2))dx$$
$$= c_1\left[2\left(\frac{x^3}{3} - \frac{x^4}{4}\right) + c_1\left(\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} - x + \frac{4x^2}{2} - \frac{4x^3}{3}\right)\right]_0^1$$
$$= c_1\left[2\left(\frac{1}{3} - \frac{1}{4}\right) + c_1\left(-1 + 2 - 1 - \frac{1}{2} + \frac{1}{5}\right)\right]$$
$$= c_1\left(\frac{1}{6} - \frac{3}{10}(1)\right) = \frac{c_1}{6} - \frac{3}{10}{c_1}^2$$
$$I = \frac{c_1}{6} - \frac{3}{10}{c_1}^2$$

Stationary values are given as

$$\frac{dI}{dc_1} = 0$$
$$\frac{1}{6} - \frac{3}{10}(2(1) = 0$$

$$\frac{1}{6} = \frac{3}{5} c_1$$

$$c_1 = \frac{5}{18} c_2 = \frac{-5}{18}$$

$$\bar{y}(x) = \frac{5}{18} x(1-x)$$

2 b) Using cauchy'sresidue Theorem evaluate. **(6)**

$$\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$$

Ans Subs $z = e^{i\theta}$

$$d\theta = \frac{dz}{iz}$$

$$\text{Cos } \theta = \frac{z^2+1}{2z}$$

$$\frac{\int \frac{dz}{iz}}{5+4\left[\frac{z^2+1}{2z}\right]} = \frac{\frac{1}{i}\int \frac{dz}{z}}{\frac{10z+4z^2+4}{2z}}$$

$$= \frac{2}{i}\int \frac{dz}{4z^2 + 10z + 4}$$

$$= \frac{1}{i}\int \frac{dz}{2z^2 + 5z + 2}$$

Roots $2z^2 + 5z + 2 = 0$

$$(2z+1)(z+2) = 0$$

$$z = -\frac{1}{2}, -2$$

Region :$|z| = 1$
unit circle

Residue at $z = \frac{-1}{2}$

$$\lim_{z\to\frac{-1}{2}} \left(z + \frac{1}{2}\right) \quad \frac{1}{i(2z+1)(z+2)}$$

$$\lim_{z\to\frac{-1}{2}} \frac{(2z+1)}{2} \frac{1}{i(2z+1)(z+2)}$$

$$\frac{1}{2i\left[\frac{-1}{2}+2\right]} = \frac{1}{2i\left[\frac{3}{2}\right]} = \frac{1}{3i}$$

$$\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta} = 2\pi i \quad \left[\frac{1}{3i}\right] = \frac{2\pi}{3}$$

2 c) Find the m. g. f of the random variable X whose p. m.f. is given by

X :	-2	3	1
P (X = x) :	**1/3**	**1/2**	**1/6**

Also find the first row moments about the ogirion. **(8)**

Ans :- By definition

$$M_0(t) = E(e^{tx}) = \sum p_i e^{tx_i} = \frac{1}{3}.e^{-2t} + \frac{1}{2}e^{3t} + \frac{1}{6}e^t$$

Now, $\mu_1{}' = \left[\frac{d}{dt}M_0(t)\right]_{t=0} = \left[-\frac{2}{3}e^{-2t} + \frac{3}{2}e^{3t} + \frac{1}{6}e^t\right]_{t=0} = -\frac{2}{3} + \frac{3}{2} + \frac{1}{6} = \frac{6}{6} = 1$

$$\mu_2{}^2 = \left[\frac{d^2}{dt^2}M_0(t)\right]_{t=0} \left[\frac{4}{3}e^{-2t} + \frac{9}{2}e^{3t} + \frac{1}{6}e^t\right]_{t=0} = \frac{4}{3} + \frac{9}{2} + \frac{1}{6} = \frac{36}{6} = 6$$

3 a) Computer $A^4 - 6A^8 + 10A^7 - 3A^6 + A + I$

Where $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ **(6)**

Ans :- Clear eqn $\lambda^3 - s_1\lambda^2 + s_2\lambda - |A| = 0$

$$S_1 = 1 + 3 + 2 = 6$$

$$s_2 = \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix}$$

$$= 6 - 0 + 2 - 3 + 3 + 2 = 10$$

$$|A| = 1[6 - 0] - 2[-2 - 1] + 3[0 - 3]$$

$$= 6 + 6 - 9 = 3$$

$$\therefore \lambda^3 - 6\lambda^2 + 10\lambda - 3 = 0$$

Replace λ by A (apply CH)

$A^3 - 6A^2 + 10A - 3I = 0$

$$A^3 - 6A^2 + 10A - 3I \overline{\big) A^9 - 6A^8 + 10A^7 - 3A^6 + A + I}$$
$$\underline{-A^9 \mathop{-}_{+} 6A^8 + 10A^7 \mathop{-}_{+} 3A^6}$$
$$A + I \quad \to \quad R$$

$$\therefore A^9 - 6A^8 + 10A^7 - 3A^6 + A + I = A^6(A^3 - 6A^2 + 10A - 3I) + (A + I)$$

But $A^3 - 6A^2 + 10A - 3I = 0$

$$\therefore A^9 - 6A^8 + 10A^7 - 3A^6 + A + I = A + I$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 \\ -1 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\therefore A^9 - 6A^8 + 10A^7 - 3A^6 + A + I = \begin{bmatrix} 2 & 2 & 3 \\ -1 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

3 b) Verify Cauchy- Schwartz inequality for the vectors $u = (-4, 2, 1)$ and v = (8, -4, -2)

(6)

Ans :-

$$\|u\| = \sqrt{(-4)^2 + 2^2 + 1^2} = \sqrt{21}$$

$$\|v\| = \sqrt{8^2 + (-4)^2 + (-2)^2} = \sqrt{84}$$

$$\|u\|\|v\| = \sqrt{21}\sqrt{84} = 42$$

$$|u - v| = |u_1 v_1 + u_2 v_2 + u_3 v_3|$$

$$= |(-4 \times 8) + (2 \times -4) + (1 \times -2)|$$

$$= |-32-8-2|$$

$$= |-42|$$

$$= 42$$

$$\therefore \ \|u\|\|v\| = |u-v|$$

It satisfies the condition of Cauchy – Schwartz inequality.

$$\therefore \ |u-v| \leq \|u\|\|v\|$$

∴ Cauchy- Schwartz inequality is verified.

3 c) Obtain Taylors and Laurent's series expansion of the function. **(8)**

$$f(z) = \frac{1}{z^2-3z+2}$$

when

(i) $|z| < 1$ (ii) $1 < |z| < 2$

Ans: $f(z) = \frac{1}{z^2-3z+2} = \frac{1}{(z-1)(z-2)}$

$$\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$$

$$1 = A(z-2) + B(z-1)$$

Subs z = 1	subs z = 2
1 = A (-1)	1 = B
A = - 1	B = 1

$$f(z) = \frac{-1}{z-1} + \frac{1}{z-2}$$

<u>**Case I :**</u>$|z| < 1$

$$f(z) = \frac{-1}{z-1} + \frac{1}{z-2}$$

$$f(z) = \frac{+1}{1-z} + \frac{1}{2\left[\frac{z-1}{2}\right]}$$

$$f(z) = \frac{1}{1-z} + \frac{1}{2\left[\frac{1-z}{2}\right]}$$

$$= (1-z)^{-1} - \frac{1}{2}\left(\frac{1-z}{2}\right)^{-1}$$

$$= \left[1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots\right] - \frac{1}{2}\left[1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + ..\right]$$

<u>**Case II:**</u> $1<|z| < 2$

$$f(z) = \frac{-1}{z-1} + \frac{1}{z-2}$$

$$= \frac{-1}{2\left[\frac{1-1}{z}\right]} + \frac{1}{2\left[\frac{z-1}{2}\right]}$$

$$= \frac{-1}{z}\left[1 - \frac{1}{z}\right]^{-1} - \frac{1}{2}\left[1 - \frac{z}{2}\right]^{-1}$$

$$= \frac{-1}{z}\left[1 + \frac{1}{z} + \left(\frac{1}{z}\right)^2 + \cdots\right] - \frac{1}{2}\left[1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \cdots\right]$$

4 a) Obtain the equation of the line of revulsion of y on x for the following data and estimate y when x = 73. (6)

X	70	72	74	76	78	80
Y	163	170	179	188	196	200

Line of Regression of y on x.

$$y = a + bx$$

$$\in y = na + b \in x$$

$$\in xy = a \in x + b \in x^2$$

x	y	X y	x^2
70	163	11410	4900
72	170	12240	5184

74	179	13246	5476
76	188	14288	5776
78	196	15288	6084
80	200	16000	6400
$\sum x = 450$	$\sum y = 1096$	$\sum xy = 82472$	$\sum x^2 = 33820$

$$\therefore \quad 1096 = 6a + 450\ b$$

82472 = 450 a + 33820 b

$$\therefore \quad a = 5.31$$

B = -212.57

$$\therefore \quad y = 5.31\ x\ - 212.57$$

Subs x = 73

Y = 5.31 (73) – 212.57

Y = 175.06

4 b) Show that the function $\int_{x_1}^{x_2} (y^2 + x^2 y')\, dx$ assumes extreme values on the straight line.

$y = x.$ **(6)**

Solution.

$F = y^2 + x^2 y'$ (x, y, y' are proved)

∴ Solution is

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$$

Diff B S by y

$$\frac{\partial F}{\partial y} = 2y$$

Diff BS by y'

$$\frac{\partial F}{\partial y'} = x^2$$

$$\therefore 2y - \frac{d}{dx}(x^2) = 0$$

$$2y - 2x = 0$$

$$y = x$$

4 c) Let R^3have Euclidian inner product use gram Schmidt process to transform the basis $\{u_1, \quad u_2, \quad u_3\}$ into orthonormal basis where , **(8)**

$\boldsymbol{u_1 = (1,0,0) u_2 = (3,7,-2) u_3 = \ (0,4,1)}$

Solution :-

Step I :-$v_1 = u_1 = (1,0,0)$

Step II :-$v_2 = u_2 - \frac{<u_2, v_1> v_1}{\|v_1\|^2}$

$$< u_2, v_1 >= \ 3 \times 1 + 7 \times 0 + -2 \times 0 = 3$$

$$\|v_1\|^2 = \sqrt{1^2 + 0^2 + 0^2} = 1$$

$$v_2 = (3, \ 7, -2) - \frac{3}{1} \ (1, \quad 0, \quad 0)$$

$$v_2 = (0, \ 7, -2)$$

Step III:-

$$v_3 = u_3 = \frac{< u_3, v_1 > v_1}{\|v_1\|^2} - \frac{< u_3, v_2 > v_2}{\|v_2\|^2}$$

$$< u_3, v_1 > = 0 + 0 + 0 = 0$$

$$< u_3, v_2 > = 0 + 28 - 2 = 26$$

$$\|v_2\|^2 = 53$$

$$v_3 = (0,4,1) - 0 - \frac{26}{53} \ (0,7,-2)$$

$$v_3 = \left(0, \frac{30}{53}, \frac{105}{53}\right)$$

Therefore,

$$v_1 = (1, 0, 0)$$

$$v_2 = (0, \ 7, -2)$$

$$v_3 = \left(0, \frac{30}{53}, \frac{105}{53}\right)$$

Forms the orthogonal Basis for R^3

Norms of me vectors

$$\|v_1\| = \sqrt{1 + 0 + 0} = 1$$

$$\|v_2\| = \sqrt{0 + 49 + 4} = \sqrt{53}$$

$$\|v_3\| = \sqrt{0 + \frac{900}{53^2} + \frac{105^2}{53^2}} = \frac{15}{\sqrt{53}}$$

∴Orthonormal Basis of R^3 are $q_1 = \frac{v_1}{\|v_1\|} = (1, 0, 0)$

$$q_2 = \frac{v_2}{\|v_2\|} = \left(\frac{0, \ 7, -2}{\sqrt{53}}\right) = \left(0, \frac{7}{\sqrt{53}}, \frac{-2}{\sqrt{53}}\right)$$

$$q_3 = \frac{v_3}{\|v_3\|} = \left(0, \frac{30}{53}, \frac{105}{53}\right) = \left(0, \frac{2}{\sqrt{53}}, \frac{7}{\sqrt{53}}\right)$$

$$\frac{15}{\sqrt{53}}$$

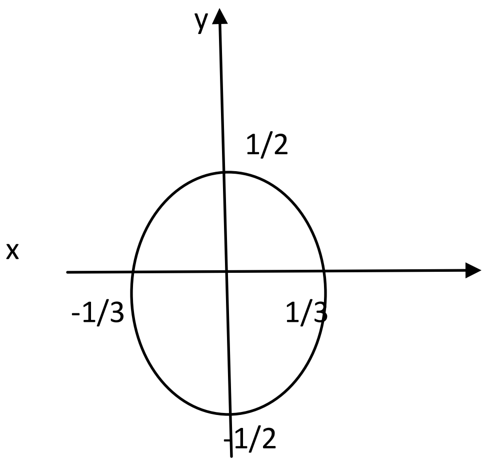
5 a)Evaluate $\int \frac{1}{z} \cos \ z \, dz$ where C is the repair $9x^2 + 4y^2 = 1$. **(6)**

Solution :-

Region $9x^2 + 4y^2 = 1$

$$\frac{x^2}{\left(\frac{1}{9}\right)} + \frac{y^2}{\left(\frac{1}{4}\right)} = 1$$

$$\frac{x^2}{\left(\frac{1}{3}\right)^2} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$$



Poles :z = 0

As per Cauchy Theorem.

$$\int \frac{cosz \;\; dz}{z} = 2\pi\, i\, f(z_0)$$

F (z) = cos z

Subs z = 0

F(0) = cos 0 = 1

$$\therefore \int \frac{cosz \;\; dz}{z} = 2\pi i\,(1) = 2\pi i$$

5 b) Seven dice are thrown 729 times. How many times do you except at least four dice to show three or five. (6)

Ans :-

Probability of getting (3 or 5) in a single toss $=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$.

This is a binomial distribution with n = 7, p = 1/3, q = 2/3.

$$\therefore \; P(X=x) = \;\; {}^{n}C_x P^x q^{n-x} = \;\; {}^{7}C_2 \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{7-x}$$

$\therefore$ P (at least 4 successes)= $p\;(x = 4, 5, 6, 7)$

$$= \ {}^7C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^3 + \ {}^7C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^2 + \ {}^7C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^1 + \ {}^7C_7\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right)^0 = \frac{379}{3^7}$$

∴ The expected number of times of getting (3 or 5) at least 4 times.

$$= Np = 729 \times \frac{379}{3^7} = 126.3$$

5 c) Show that the matrix A = $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ **is diagonalisable.** **(8)**

Find the diagonal from D and the diagonalising matrix M.

Ans :-The characteristic equation is

$$\begin{bmatrix} -9-\lambda & 4 & 4 \\ -8 & 3-\lambda & 4 \\ -16 & 8 & 7-\lambda \end{bmatrix} = 0$$

$$\therefore \quad (1+\lambda)(1+\lambda)(3-\lambda) = 0 \qquad \therefore \lambda = -1, -1, 3.$$

(i) For $\lambda = -1, [A - \lambda, I]\, X = 0$ gives

$$\begin{bmatrix} -8 & 4 & 4 \\ -8 & 4 & 4 \\ -16 & 8 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By $R_2 - R_1$

$R_3 - 2R_1$

$$-(1/4)R_1 \begin{bmatrix} 2 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad 2x_1 - x_2 - x_3 = 0$$

The rank of the coefficient matrix is 1. The number of unknowns is 3.

Hence, there are 3 - 1 = 2 linearly independent solutions.

Putting $x_2 = 2t$ and $x_3 = 2s$, we get $2x_1 = x_2 + x_3$

$$\therefore \quad 2x_1 = 2t + 2s \qquad \therefore \quad x_1 = t + s$$

$$\therefore \quad X_1 = \begin{bmatrix} s & + & t \\ 0 & + & 2t \\ 2s & + & 0 \end{bmatrix} = s\begin{bmatrix}1\\0\\2\end{bmatrix} + t\begin{bmatrix}1\\2\\0\end{bmatrix}$$

∴ Corresponding to the eigen values -1, we get the following two linearly independent eigenvectros.

$$X_1 = [1, 0, 2]' \qquad and \qquad X_2 = [1, \quad 2, \quad 0]'$$

(i) For $\lambda = 3, [A - \lambda_2 I]\, X = 0$ gives

$$\begin{bmatrix} -12 & 4 & 4 \\ -8 & 0 & 4 \\ -16 & 8 & 4 \end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

By $R_2 - R_1$

$R_3 - R_1$

$$\begin{bmatrix} -12 & 4 & 4 \\ 4 & -4 & 0 \\ -4 & 4 & 0 \end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

By $R_3 + R_2$ $\begin{bmatrix} -12 & 4 & 4 \\ 4 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

By $-(1/4)R_1$

$(1/4)R_2$

$$\begin{bmatrix} 3 & -1 & -1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

∴ $3x_1 - x_2 - x_3 = 0$and$x_1 - x_2 = 0$ ∴ $x_1 = x_2$

Putting $x_2 = t$, we get $x_1 = t$ and $x_3 = 3x_1 - x_2 = 3t - t = -2t$

$$\therefore \; x_3 = \begin{bmatrix} t \\ t \\ 2t \end{bmatrix} = t\begin{bmatrix}1\\1\\2\end{bmatrix}$$

∴ Corresponding to eigen value 3, we get the following eigen vector.

$$X_3 = [1, \; 1, 2]$$

Although eigen values of A are not distinct the geometric multiplicity of each eigen value is equal to its algebraic multiplicity, A is diagnosable.

Since, $M^{-1} \quad AM = D$, the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ will be diagonalised to

the diagonal matrix D $= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ by the transforming matrix M $= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 0 & 2 \end{bmatrix}$

6 a) Example on Mean and Variance of Continuous Probability Distribution.

Example 1 : A continuous random variable X has the p. d. f. defined by $f(X) = A + BX$, $0 \le X \le 1$.The mean of the distribution is 1/3, find A and B. **(6)**

Ans :- Since f (x) is a probability distribution $\int_{-\infty}^{\infty} f(x)dX = 1$

By deta$\int_0^1 (A + BX)dX = 1 \therefore \left[AX + \frac{BX^2}{2}\right]_0^1 = 1 \qquad \therefore A + \frac{B}{2} = 1$ ……………..(i)

Since the mean is $\frac{1}{3}$. $\int_0^1 \; Xf(X)\,dX = \frac{1}{3} \qquad \therefore \int_0^1 (A + BX)XdX = \frac{1}{3}$

$$\therefore \int_0^1 (AX + BX^2)dx = \frac{1}{3} \quad \therefore \left[A\frac{x^2}{2} + \frac{Bx^3}{3}\right]_0^1 = \frac{1}{3}$$

$\therefore \frac{A}{2} + \frac{B}{3} = \frac{1}{3} \qquad \therefore 3A + 2B = 2$ ……………….(ii)

Solving the equations (i) and (ii), we get A = 2, B = – 2

$\therefore$ **The p. d. f. is $f(X) = 2 - 2x$, $0 \le X \le 1$.**

6 b) Find e^A and 4^A if A $=\begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$ **(6)**

The characteristic equation of A is

$$\begin{vmatrix} (3/2 - \lambda) & 1/2 \\ 1/2 & (3/2 - \lambda) \end{vmatrix} = 0$$

$$\therefore\ \left(\frac{3}{2}-\lambda\right)^2-\frac{1}{4}=0 \qquad \therefore\ \frac{9}{4}-3\lambda+\lambda^2-\frac{1}{4}=0$$

$$\therefore\ \lambda^2-3\lambda\ +2=0 \qquad \therefore\ (\lambda-1)(\lambda-2)=0$$

$$\therefore\ \lambda=1,2.$$

(i) For $\lambda=1, [A-\lambda I]X=0$ gives

$$\begin{bmatrix}1/2 & 1/2\\ 1/2 & 1/2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

By $2R_1$

$$2R_2\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

By $R_2-R_1\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$$\therefore\ X_1+X_2=0$$

Putting $x_2=-t$, we get $x_1=t$

$$\therefore\ X_1=\begin{bmatrix}t\\ -t\end{bmatrix}=t\begin{bmatrix}1\\ -1\end{bmatrix}$$

Hence, the eigen vector is $[1,-1]'$

(i) For $\lambda=2, [A-\lambda I]X=0$ gives

$$\begin{bmatrix}-1/2 & 1/2\\ 1/2 & -1/2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

By $2R_1$

$$2R_2\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

By $R_2+R_1\begin{bmatrix}-1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$

$$\therefore\ -X_1+X_2=0 \qquad \therefore\ X_1=X_2$$

Putting $x_2=t, x_1=t$, we get

$$\therefore \quad X_2 = \begin{bmatrix} t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Hence, the eigen vector is $[1, 1]'$

$\therefore$ M = $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ $\therefore \quad |M| = 2$

$$M^{-1} = \frac{adj.M}{|M|} = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Now D = $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$$If\ f\ (A) = e^A, \qquad f\ (D) = e^D = \begin{bmatrix} e^1 & 0 \\ 0 & e^2 \end{bmatrix}$$

$$If\ f\ (A) = 4^A, \qquad f\ (D) = 4^D = \begin{bmatrix} 4^1 & 0 \\ 0 & 4^2 \end{bmatrix}$$

$$\therefore\ e^A = Mf(D)M^{-1} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e & 0 \\ 0 & e^2 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} e & e^2 \\ -e & e^2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\therefore\ e^A = \frac{1}{2}\begin{bmatrix} e & + & e^2 - e & + & e^2 \\ -e & + & e^2\ e & + & e^2 \end{bmatrix}$$

6 c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 \ dx}{(x^2+a^2)(x^2+b^2)}$ **(8)**

(ii) Now, $zf(z) = \frac{z^3}{(z^2+a^2)(z^2+b^2)} \rightarrow$ 0 as $|z| \rightarrow \infty$

(iii) Now , $(z^2 + a^2)(z^2 + b^2) = 0\ \ i.e. z = +ai, -ai, +bi, -bi.\ \ Of\ these\ z = ai,$ $z = bi$lie in the upper to the z- plane.

(iv) Residue (at z = ai) =$\lim_{z \to ai}$ $(z - ai).\ \frac{z^2}{(z-ai)(z+ai)(z^2+b^2)}$

$$= \frac{-a^2}{2ai(-a^2 + b^2)} = \frac{a}{2i(a^2 - b^2)}$$

Similarly, Residue (at z = bi) $= \frac{-b^2}{2bi(a^2-b^2)} = \frac{-b}{2i(a^2-b^2)}$

(v) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)}\ dx = 2\pi\ i\ \left[\frac{a}{2i(a^2-b^2)} + \frac{-b}{2i(a^2-b^2)}\right]$

$$= \frac{\pi}{a+b}$$